25 Practice Problems for Derivatives

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1 Power rule

Find the derivative of each of the following functions:

1. $a(x) = 3x^7 - 10x + 24x^3 - 1$
2. $b(x) = \frac{x^2-5x+3}{\sqrt[3]{x}}$
3. $c(x) = 5^2 - \frac{3}{x^4}$

2 Product rule

Find the derivative of each of the following functions:

1. $d(x) = \sin(x)\cos(x)$
2. $e(x) = x^{10}e^x$
3. $f(x) = 2^x x^2 \tan(x)$

3 Quotient rule

Find the derivative of the following function:

- $g(x) = \frac{2e^x+4}{\cos(x)+3x-1}$

Find the <u>second</u> derivative of the following function:

- $h(x) = \frac{\sin(x)}{x^2}$

4 Chain rule

For each of the following, write the given function as a composition of two functions, i.e., as $f(g(x))$, where you have identified f and g. Then take the derivative using the chain rule. Note: some problems may require more than one chain rule.

1. $i(x) = (22x^4 + \sqrt{x})^9$
2. $j(x) = \sin(x^3 + 1)$
3. $k(x) = \sin^3(x) + 1$
4. $l(x) = \ln(\sin(x))$
5. $m(x) = \sin\left(\cos\left(e^{5x^2-3x+2}\right)\right)$

5 Multiple rules

Find the derivative of each of the following functions:

1. $n(x) = \frac{x\ln(x)}{x^{3/2}+1}$
2. $o(x) = (3x^5 - 2x + 7)^{13}e^x$
3. $p(x) = \ln\left(x - \frac{1}{e^x}\right)$
4. $q(x) = \ln(x\sin(x))$

6 Implicit differentiation

Find y' in each of the following examples. Remember, y **is a function**! This means you must use some extra derivative rules. Golden rule: <u>if your derivative of a $y-$term doesn't have $y^{'}$, you missed a derivative rule</u>!

1. $x^2y + \sin(y) = 5y^2 + 3$
2. $e^{2y+1} = x$

Find $y^{''}$ from the following equation.

- $x^3 + y^3 = 1$

7 Logarithmic differentiation

In the following problems you will find it helpful to make an equation of the form $y = ...$ and take a natural logarithm of both sides before differentiating.

1. $r(x) = x^x$
2. $s(x) = (x^2 - 4)^{\sin(x)}$
3. $$t(x) = \frac{\sqrt{4x^3 - x + 1}}{x^{2/3}\tan(x)}$$

8 Related rates

1. A spherical snowball is melting in the sun. Its radius is decreasing at a rate of 1 cm/s. When the radius reaches 5 cm, how quickly is the snowball losing volume?
2. An airplane flies overhead 2 miles up at a speed of 500 mi/hr. When it has travelled 1 mile from where you are, how quickly is the distance from you to the airplane increasing?